RMSC 4003
Statistical Modeling in Financial Markets
Tutorial 1 Solution

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September 11, 2014

Example 0.1. Show that

(a) $\mathrm{E}(aX+b) = a\,\mathrm{E}(X) + b$.

(b) $\mathrm{E}(X+Y) = \mathrm{E}(X) + \mathrm{E}(Y)$.

(c) $\mathrm{E}(XY) = \mathrm{E}(X)\,\mathrm{E}(Y)$ if X and Y are independent.

Proof. (a)

$$\begin{aligned}
\mathrm{E}(aX+b) &= \int (ax+b) f(x) dx \\
&= a \int x f(x) dx + b \int f(x) dx \\
&= a\,\mathrm{E}(X) + b.
\end{aligned}$$

(b)

$$\begin{aligned}
\mathrm{E}(X+Y) &= \int\int (x+y) f(x,y) dx dy \\
&= \int\int x f(x,y) dx dy + \int\int y f(x,y) dx dy \\
&= \int\int x f(x,y) dy dx + \int y \int f(x,y) dx dy \\
&= \int x \int f(x,y) dy dx + \int y f(y) dy \\
&= \int x f(x) dx + \int y f(y) dy \\
&= \mathrm{E}(X) + \mathrm{E}(Y).
\end{aligned}$$

(c)

$$
\begin{aligned}
\mathrm{E}(XY) &= \int\int xyf(x,y)dxdy \\
&= \int\int xyg(x)h(y)dxdy \\
&= \int yh(y)\int xg(x)dxdy \\
&= \int yh(y)\,\mathrm{E}(X)dy \\
&= \mathrm{E}(X)\int yh(y)dy \\
&= \mathrm{E}(X)\,\mathrm{E}(Y).
\end{aligned}
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$\square$